Risk and Return I
(Calculating Returns)

Learning Objectives

1. Calculate and interpret dividend/current yield, capital gains yield, holding period return, arithmetic average return, and geometric average return
2. Identify settings when the arithmetic or geometric average is a more appropriate estimate of expected future returns

Returns

Dollar Return = Dividend + Δ in Market Value

$$\text{Percentage Return} = \frac{\text{dollar return}}{\text{beginning value}}$$
$$= \frac{\text{dividend} + \Delta \text{ in value}}{\text{beginning value}}$$
$$= \frac{\text{dividend}}{\text{beginning value}} + \frac{\Delta \text{ in value}}{\text{beginning Value}}$$
$$= \text{dividend yield} + \text{capital gains yield}$$
$$= \frac{\text{Total ending value}}{\text{beginning value}} - 1$$

Returns: Example

You bought 100 shares of Wal-Mart (WMT) one year ago today at $25. Over the last year, you received 20 cents per share in dividends, and at the end of the year, the stock sells for $30. What are your dollar and % returns, dividend yield, and capital gains yield?

Answer:

- Dollar Return: 20 + (3,000 – 2,500) = 520.
- Percentage return: 520/2,500 = 20.8%
- Dividend Yield: 20/2,500 = 0.08%
- Capital Gains Yield = (3,000–2,500)/2,500 = 20%

Holding-Period Return

The holding period return (HPR) is the total % return that an investor would get over a period of *n* years. If the return during year *i* is given as r_i: the HPR is calculated as

$$HPR = \prod_{i=1}^{n}(1+r_i) - 1$$
$$= (1+r_1)(1+r_2)(...)(1+r_n) - 1$$

Holding-Period Return: Example

Your investment gives the following returns over a four-year period. What is your holding period return?

Year	Return
1	0.1
2	-0.05
3	0.2
4	0.15

$$HPR = \prod_{i=1}^{n}(1+r_i) - 1$$
$$= (1.1)(0.95)(1.20)(1.15) - 1$$
$$= 1.4412 - 1 = 44.12\%$$

Arithmetic Mean Return

The Arithmetic Mean Return (AMR) is the return earned in an average period (calculated over a number of periods) If the return during year *i* is given as r_i the Arithmetic Mean Return is calculated as

$$AMR = \frac{\sum_{i=1}^{n} r_i}{n}$$

$$\frac{(r_1 + r_2 + \ldots + r_n)}{n}$$

Arithmetic Mean Return: Example

Using the returns from the previous example, what is the Arithmetic Mean Return?

Year	Return
1	0.1
2	-0.05
3	0.2
4	0.15

$$AMR = \frac{\sum_{i=1}^{n} r_i}{n}$$

$$= \frac{0.10 + (-0.05) + 0.20 + 0.15}{4}$$

$$= 0.10 = 10\%$$

Geometric Mean Return

The Geometric Mean Return (GMR) is the average *compound return* per period (calculated over a number of periods). It is (basically) the compound return that results in the Holding Period Return. If the return during year *i* is given as r_i the Geometric Mean Return is calculated as

$$GMR = \left(\prod_{i=1}^{n} (1 + r_i) \right)^{1/n} - 1$$

$$= \left((1 + r_1)(1 + r_2)(\ldots)(1 + r_n) \right)^{1/n} - 1$$

$$= \left(1 + HPR \right)^{1/n} - 1$$

Geometric Mean Return: Example

Using the returns from the previous example, what is the Geometric Mean Return?

Year	Return
1	0.1
2	-0.05
3	0.2
4	0.15

$$GMR = (1.1\ x\ 0.95\ x\ 1.20\ x\ 1.15)^{1/4} - 1$$
$$= (1.4412)^{1/4} - 1$$
$$= 0.0958 = 9.58\%$$

Arithmetic vs. Geometric Mean

- The geometric average will always be closer to zero than the arithmetic average unless all the returns are equal
- The "spread" between the two increases with the volatility (variance) of the returns
- Which is better?
  - The arithmetic average is the best estimate of returns for a ***single*** period.
  - The geometric average is the best estimate of annual returns for a ***multi-year*** period.

Recap

1. Calculating dividend yield, capital gains yield, holding period return, and total return
2. Holding period return is the ***total*** % return over some period
3. Arithmetic mean return is the "usual" average
4. Geometric mean return is the compound periodic return over the holding period
5. Geometric mean return is closer to zero than arithmetic mean return (spread depends on volatility)
6. Use arithmetic mean for single-period estimates and geometric mean for multiple-period estimates

Test Yourself

A stock has an initial price of $50. It pays dividends each year and has a price pattern as noted in the following table. All dividends are paid at the end of the year. Calculate (1) the Dividend Yield, Capital Gains Yield, and Total Returns for each year, (2) the Holding period return over the three years (assume all cash flows are reinvested each year), (3) the Arithmetic Mean Return over the three years, and (4) the Geometric Mean Return over the three years

Year	Price (EOY)	Dividend	Dividend Yield	Capital Gains Yield	Total Return
1	54	2			
2	59.40	2.50			
3	56.43	1.00			

Test Yourself: Solution (1)

(1) Calculate the Dividend Yield, Capital Gains Yield, and Total Returns for each year

Yr	Price (EOY)	Div	Div. Yield	Capital Gains Yield	Total Return
1	54	2	2/50=0.04	(54-50)/50 = 0.08	0.04+0.08=0.12
2	59.40	2.50	2.50/54=0.0463	(59.40-54)/54=0.10	0.0463+0.10=0.1463
3	56.43	1.00	1/59.40=0.0168	(56.43-59.40)/59.40= -0.05	0.0168-0.05= -0.0332

Test Yourself: Solution (2)

(2) Calculate the Holding Period Return return over the three years (assume all cash flows are reinvested each year)

Yr	Price (EOY)	Div	Div. Yield	Capital Gains Yield	Total Return
1	54	2	2/50=0.04	(54-50)/50 = 0.08	0.04+0.08=0.12
2	59.40	2.50	2.50/54=0.0463	(59.40-54)/54=0.10	0.0463+0.10=0.1463
3	56.43	1.00	1/59.40=0.0168	(56.43-59.40)/59.40= -0.05	0.0168-0.05= -0.0332

$$\text{Holding Period Return} = (1.12)(1.1463)(0.9668) - 1 = 24.12\%$$

Test Yourself: Solution (3)

(3) Calculate the Arithmetic and Geometric Mean Returns over the three years

Yr	Price (EOY)	Div	Div. Yield	Capital Gains Yield	Total Return
1	54	2	2/50=0.04	(54-50)/50 = 0.08	0.04+0.08=0.12
2	59.40	2.50	2.50/54=0.0463	(59.40-54)/54=0.10	0.0463+0.10=0.1463
3	56.43	1.00	1/59.40=0.0168	(56.43-59.40)/59.40= -0.05	0.0168-0.05= -0.0332

$$Arithmetic\ Mean = \frac{0.12 + 0.1463 + (-0.0332)}{3} = 7.77\%$$

$$Geometric\ Mean = ((1.12)(1.1463)(0.9668))^{1/3} - 1 = (1.2412)^{0.333} - 1 = 7.47\%$$